

Higgs-Dilaton Cosmology: an effective field theory approach

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The Higgs-Dilaton cosmological model is able to describe simultaneously an inflationary expansion in the early Universe and a dark energy dominated stage responsible for the present day acceleration. It also leads to a non-trivial relation between the spectral tilt of scalar perturbations n_s and the dark energy equation of state ω . We study the self-consistency of this model from an effective field theory point of view. Taking into account the influence of the dynamical background fields, we determine the effective cut-off of the theory, which turns out to be parametrically larger than all the relevant energy scales from inflation to the present epoch. We finally formulate the set of assumptions needed to estimate the amplitude of the quantum corrections in a systematic way and show that the connection between n_s and ω remains unaltered if these assumptions are satisfied.

I. INTRODUCTION

The shortcomings of the hot big bang model can be solved in an elegant way if we assume that the Universe underwent an inflationary period in its early stages. The easiest way for this paradigm to be realized is by a scalar field slowly rolling down towards the minimum of its potential [1].

As discussed in Ref. [2], inflation does not necessarily require the existence of a new degree of freedom. The role of the inflaton can be played by the Standard Model (SM) Higgs field with its mass lying in the interval where the SM can be considered a consistent effective field theory up to the inflationary scale. More precisely, if the Higgs boson is non-minimally coupled to gravity and the value of the corresponding coupling constant ξ_h is sufficiently large, the model is able to provide a successful inflationary period followed by a graceful exit to the standard hot Big Bang theory [3, 4]. The implications of this scenario have been extensively studied in the literature [5–20]. Earlier studies of non-minimally coupled scalar fields in the context of inflation can be also found in Refs. [21–23].

When the Higgs inflation model described above is rewritten in the so-called Einstein frame, where the gravity part takes the usual Einstein-Hilbert form, it becomes essentially non-polynomial and thus non-renormalizable, even if the gravity part is dropped off. Therefore, it should be understood as an effective field theory valid only up to a certain cut-off energy scale Λ . The usual criterion for determining the cut-off of the theory is based on the violation of tree level unitarity in high-energy scattering processes. As discussed in Refs. [8, 9, 16, 17] the tree-level scattering amplitudes above the *electroweak vacuum* appear to hit the perturbative unitarity bound at energies $\Lambda \sim M_P/\xi_h$. At that scale perturbation theory breaks down. Whether the theory enters into the non-perturbative strong-coupling regime or it requires an ultraviolet completion at higher energies is still an open question. Nevertheless, the Higgs inflation scenario is self-consistent. As shown in Ref. [24] (see also [25]), the proper cutoff of the theory depends on the *dynamical* expectation value of the Higgs field, which makes the theory weakly coupled for all the relevant energy scales in the evolution of the Universe. In other words, the SM with a large non-minimal coupling of the Higgs field to gravity represents a viable effective theory for the description of inflation, reheating, and the hot Big Bang theory.

The Higgs inflation scenario can be easily incorporated into a larger framework, the Higgs-Dilaton model [26, 27]. The key element of this extension is scale-invariance (SI). No dimensional parameters such as masses are allowed to appear in the action. All the scales are instead induced by the spontaneous breaking of SI. This is achieved by the introduction of a new scalar degree of freedom -the dilaton-, which becomes the Goldstone boson of the broken symmetry and remains exactly massless. The coupling of the dilaton field to matter is weak and takes place only through derivative couplings, not contradicting therefore any 5th force experimental bounds [28].

Although the dilatation symmetry described above forbids the introduction of a cosmological constant term, the ever-present cosmological constant problem reappears associated to the fine-tuning of the dilaton self-interaction [26]. However, if the dilaton self-coupling β is chosen to be zero (or required to vanish due to some yet unknown reason), a

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slight modification of general relativity (GR), known as Unimodular Gravity (UG), provides a dynamical dark energy (DE) stage in good agreement with observations. The scale-invariant UG gives rise to a “run-away” potential for the dilaton [26], which plays the role of a quintessence field. The strength of such a potential is determined by an integration constant that appears in the Einstein equations of motion due to the unimodular constraint $\hat{g} = -1$ on the metric determinant. The common origin of the inflationary and DE dominated stages in Higgs-Dilaton inflation allowed to derive extra bounds on the initial inflationary conditions¹, as well as potentially testable relations between the early and late Universe observables [27].

Our purpose here is to study, following the approach of Ref. [24], the self-consistency of the Higgs-Dilaton model by adopting an effective field theory point of view. We will estimate the field-dependent cut-offs associated to the different interactions among scalars fields, gravity, vector bosons and fermions. We will identify the lowest cut-off as a function of the background fields and show that its value is higher than the typical energy scales describing the Universe during its different epochs. The issue concerning quantum corrections generated by the loop expansion is also addressed. Since the model is non-renormalizable, an infinite number of counter-terms must be added in order to absorb the divergences. We will adopt a “minimal setup” that keeps intact the exact and approximative symmetries of the classical action and does not introduce any extra degrees of freedom. Within this approach, the relations connecting the inflationary and the dark energy domination periods hold even in the presence of quantum corrections.

The structure of the paper is as follows. In Section II we briefly review the Higgs-Dilaton model. In Section III we calculate the cut-off of the theory in the Jordan frame and compare it with the other relevant energy scales in the evolution of the Universe. In Section IV we propose a “minimal setup” which removes all the divergences and discuss the sensitivity of the cosmological observables to radiative corrections. Section V contains the conclusions.

II. HIGGS-DILATON COSMOLOGY

We start by reviewing the main results of Refs. [26, 27], where the Higgs-Dilaton model was proposed and studied in detail. The two main ingredients of the theory are outlined below. The first one is the invariance of the SM action under global scale transformations, which leads to the absence of any dimensional parameters or scales. Denoting by $\Phi(x)$ the field content of the theory in a metric $g_{\mu\nu}(x)$, these transformations can be written as²

$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x) , \quad \Phi(x) \rightarrow \sigma^{d_\Phi} \Phi(\sigma x) , \quad (2.1)$$

with σ^{d_Φ} the so-called scaling dimension and σ an arbitrary constant. In order to achieve invariance under these transformations, we let the masses and dimensional couplings in the theory to be dynamically induced by a field. The simplest choice would be to use the SM Higgs, already present in the theory. Note however that this option is clearly incompatible with the experiment. As discussed in Refs. [22, 29], the excitations of the Higgs field in this case become massless and completely decoupled from the SM particles.

The next simplest possibility is to introduce a new scalar singlet under the SM gauge group. We will refer to it as the dilaton χ . The coupling between the new field and the SM particles, with the exception of the Higgs boson, is forbidden by quantum numbers. The corresponding Lagrangian is given by

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi)R + \mathcal{L}_{\text{SM}[\lambda \rightarrow 0]} - \frac{1}{2}g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi, \varphi) , \quad (2.2)$$

where φ is the SM Higgs field doublet and $\xi_h \sim 10^3 - 10^5$, $\xi_\chi \sim 10^{-3}$, are respectively the non-minimal couplings of the Higgs and dilaton fields to gravity [27]. The term $\mathcal{L}_{\text{SM}[\lambda \rightarrow 0]}$ is the SM Lagrangian without the Higgs potential, which in the present scale-invariant theory becomes

$$V(\chi, \varphi) = \lambda \left(\varphi^\dagger \varphi - \frac{\alpha}{2\lambda} \chi^2 \right)^2 + \beta \chi^4 , \quad (2.3)$$

with λ the self-coupling of the Higgs field.

In order for this theory to be phenomenologically viable, we demand the existence of a symmetry-breaking ground state with non-vanishing background expectation value for both³ the dilaton ($\bar{\chi}$) and the Higgs field in the unitary

¹ The fine-tuning needed to reproduce the present dark energy abundance is transferred into the initial inflationary conditions for the fields at the beginning of inflation.

² For a theory invariant under all diffeomorphisms, this is equivalent to

$$g_{\mu\nu}(x) \rightarrow \sigma^{-2} g_{\mu\nu}(x) , \quad \Phi(x) \rightarrow \sigma^{d_\Phi} \Phi(x) .$$

³ If $\bar{\chi} = 0$ the Higgs field is massless, and if $\bar{h} = 0$ there is no electroweak symmetry breaking.

gauge (\bar{h}) . This is given by

$$\bar{h}^2 = \frac{\alpha}{\lambda} \bar{\chi}^2 + \frac{\xi_h}{\lambda} R, \quad \text{with} \quad R = \frac{4\beta\lambda}{\lambda\xi_\chi + \alpha\xi_h} \bar{\chi}^2. \quad (2.4)$$

All the physical scales are proportional to the non-zero background value of the dilaton field. For instance, the SM Higgs mass is given by

$$m_H^2 = 2\alpha M_P^2 \frac{(1 + 6\xi_\chi) + \frac{\alpha}{\lambda}(1 + 6\xi_h)}{(1 + 6\xi_\chi)\xi_\chi + \frac{\alpha}{\lambda}(1 + 6\xi_h)\xi_h} + \mathcal{O}(\beta), \quad (2.5)$$

with $M_P^2 \equiv \xi_h \bar{h}^2 + \xi_\chi \bar{\chi}^2 \propto \bar{\chi}^2$ the effective Planck scale in the Jordan frame. The same happens with the effective cosmological constant

$$\Lambda = \frac{1}{4} M_P^2 R = \frac{\beta M_P^4}{(\xi_\chi + \frac{\alpha}{\lambda}\xi_h)^2 + 4\frac{\beta}{\lambda}\xi_h^2}, \quad (2.6)$$

which depending on the value of the dilaton self-coupling β , gives rise to a flat ($\beta = 0$), deSitter ($\beta > 0$) or anti-deSitter ($\beta < 0$) spacetime. It is important to notice however that physical observables, corresponding to dimensionless ratios between scales or masses, are independent of the particular value of the background field $\bar{\chi}$. In order to reproduce the ratio between the different energy scales, the parameters of the model must be properly fine-tuned. As shown in Eq. (2.5), the difference between the electroweak and the Planck scale is encoded in the parameter⁴ $\alpha \sim 10^{-35} \lll 1$. Similarly, the hierarchy between the cosmological constant and the electroweak scale, cf. Eq. (2.6), implies $\beta \lll \alpha$. The smallness of these parameters, together with the tiny value of the non-minimal coupling ξ_χ , gives rise to an approximate shift symmetry for the dilaton field at the classical level, $\chi \rightarrow \chi + \text{const.}$ As we will show in Section IV, this fact will have important consequences for the analysis of the quantum effects.

The second ingredient of the Higgs-Dilaton cosmological model is the replacement of GR by Unimodular Gravity, which is just a particular case of the set of theories invariant under transverse diffeomorphisms (TDiff). These theories generically contain an extra scalar degree of freedom on top of the massless graviton (for a general discussion see for instance Ref. [30] and references therein). In UG the number of dynamical components of the metric is effectively reduced to the standard value by requiring the metric determinant \hat{g} to take some fixed constant value, conventionally $|\hat{g}| = 1$. As shown in Ref. [26], the equations of motion of a theory subject to that constraint

$$\mathcal{L}_{\text{UG}} = \mathcal{L}[\hat{g}_{\mu\nu}, \partial\hat{g}_{\mu\nu}, \Phi, \partial\Phi], \quad (2.7)$$

coincide with those obtained from a diffeomorphism invariant theory (Diff) with modified action

$$\frac{\mathcal{L}_{\text{Diff}}}{\sqrt{-g}} = \mathcal{L}[g_{\mu\nu}, \partial g_{\mu\nu}, \Phi, \partial\Phi] + \Lambda_0. \quad (2.8)$$

Note that, from the point of view of UG, the parameter Λ_0 is just a conserved quantity associated to the unimodular constraint and it should not be understood as a cosmological constant.

Since the two formulations are completely equivalent⁵, we will stick to the Diff invariant language. Expressing the theory resulting from the combination of the above ideas in the unitary gauge $\varphi^T = (0, h/\sqrt{2})$ we get

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}(\xi_\chi \chi^2 + \xi_h h^2)R - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}(\partial h)^2 - U(\chi, h), \quad (2.9)$$

where the potential includes now the UG integration constant Λ_0

$$U(\chi, h) \equiv V(\chi, h) + \Lambda_0 = \frac{\lambda}{4} \left(h^2 - \frac{\alpha}{\lambda} \chi^2 \right)^2 + \beta \chi^4 + \Lambda_0. \quad (2.10)$$

The phenomenological consequences of Eq. (2.9) are more easily discussed in the Einstein frame. Let us then perform a conformal redefinition of the metric $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with conformal factor $\Omega^2 = M_P^{-2}(\xi_\chi \chi^2 + \xi_h h^2)$. Using the standard relations [31]

$$\sqrt{-g} = \Omega^{-4} \sqrt{\tilde{g}} \quad \text{and} \quad R = \Omega^2 \left(\tilde{R} + 6\tilde{\square} \ln \Omega - 6\tilde{g}_{\mu\nu} \partial_\mu \ln \Omega \partial_\nu \ln \Omega \right), \quad (2.11)$$

⁴ Note that the alternative choice $\xi_h \gg 1$ is not compatible with CMB observations, cf. Eq. (2.24) and Fig. 5.

⁵ As usual, there are some subtleties related to the quantum formulation of (unimodular) gravity. However, these will not play any role in the further developments. The interested reader is referred to the discussion in Ref. [30] and references therein.

we get

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \tilde{K}(\chi, h) - \tilde{U}(\chi, h) , \quad (2.12)$$

where

$$\tilde{U}(\chi, h) \equiv \frac{U(\chi, h)}{\Omega^4} \equiv \frac{M_P^4}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \left[\frac{\lambda}{4} \left(h^2 - \frac{\alpha}{\lambda} \chi^2 \right)^2 + \beta \chi^4 + \Lambda_0 \right] , \quad (2.13)$$

is the potential (2.10) in the new frame. The non-canonical kinetic term in Eq. (2.12) can be written as

$$\tilde{K}(\chi, h) = \kappa_{ij}^E \tilde{g}^{\mu\nu} \partial_\mu \Phi^i \partial_\nu \Phi^j , \quad (2.14)$$

where the quantity

$$\kappa_{ij}^E \equiv \frac{1}{\Omega^2} \left(\delta_{ij} + \frac{3}{2} M_P^2 \frac{\partial_i \Omega^2 \partial_j \Omega^2}{\Omega^2} \right) \quad (2.15)$$

can be interpreted as the metric in the two-dimensional field space $(\Phi^1, \Phi^2) = (\chi, h)$ in the Einstein-frame. Note that, unlike the simplest Higgs inflationary scenario [2], Eq. (2.14) cannot be recast in canonical form by field redefinitions. In fact, the Gaussian curvature associated to (2.15) does not identically vanish unless $\xi_h = \xi_\chi$, which, as shown in Ref. [27], is not consistent with observations. Nevertheless, it is possible to write the kinetic term in a quite simple diagonal form. As shown in Ref. [27], the whole inflationary period takes place inside a field space domain in which the contribution of the integration constant Λ_0 is completely negligible. We will refer to this domain as the “scale invariant region” and assume that it is maintained even when the radiative corrections are taken into account (cf. Section IV). In this case, the dilatational Noether’s current in the slow-roll approximation, $(1 + 6\xi_\chi)\chi^2 + (1 + 6\xi_h)h^2$, is approximately conserved, which suggests the definition of the set of variables

$$\rho = \frac{M_P}{2} \log \left[\frac{(1 + 6\xi_\chi)\chi^2 + (1 + 6\xi_h)h^2}{M_P^2} \right] , \quad \tan \theta = \sqrt{\frac{1 + 6\xi_h}{1 + 6\xi_\chi}} \frac{h}{\chi} . \quad (2.16)$$

The physical interpretation of these variables is straightforward. They are simply adequately rescaled polar variables in the (h, χ) plane. Expressed in terms of ρ and θ , the kinetic term (2.14) turns out to be

$$\tilde{K} = \left(\frac{1 + 6\xi_h}{\xi_h} \right) \frac{1}{\sin^2 \theta + \varsigma \cos^2 \theta} (\partial \rho)^2 + \frac{M_P^2 \varsigma}{\xi_\chi} \frac{\tan^2 \theta + \eta}{\cos^2 \theta (\tan^2 \theta + \varsigma)^2} (\partial \theta)^2 , \quad (2.17)$$

with

$$\eta = \frac{\xi_\chi}{\xi_h} \quad \text{and} \quad \varsigma = \frac{(1 + 6\xi_h)\xi_\chi}{(1 + 6\xi_\chi)\xi_h} . \quad (2.18)$$

The potential (2.13) is naturally divided into a scale-invariant part, depending only on the θ field, and a scale-breaking part, proportional to Λ_0 and depending on both θ and ρ . These are respectively given by

$$\tilde{U}(\theta) = \frac{\lambda M_P^4}{4\xi_h^2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + \varsigma \cos^2 \theta} \right)^2 , \quad \tilde{U}_{\Lambda_0}(\rho, \theta) = \Lambda_0 \left(\frac{1 + 6\xi_h}{\xi_h} \right)^2 \frac{e^{-4\rho/M_P}}{(\sin^2 \theta + \varsigma \cos^2 \theta)^2} , \quad (2.19)$$

where we have safely neglected the contribution of α and β in Eq. (2.13). Note that the non-minimal couplings of the fields to gravity with $\Lambda_0 > 0$ naturally generate a “run-away” potential for the physical dilaton, similar to those considered in the pioneering works on quintessence [32–34].

The inflationary period of the expansion of the Universe takes place for field values $\xi_h h^2 \gg \xi_\chi \chi^2$. From the definition of the angular variable θ in Eq. (2.16), this corresponds to⁶ $\tan^2 \theta \gg \eta$. In that limit, we can neglect the η term in the kinetic term (2.17) and perform an extra field redefinition

$$r = \gamma^{-1} \rho \quad \text{and} \quad |\phi'| = \frac{M_P}{a} \tanh^{-1} [\sqrt{1 - \varsigma} \cos \theta] , \quad (2.20)$$

⁶ Strictly speaking, the condition $\tan^2 \theta \gg \eta$ holds beyond the inflationary region $\xi_h h^2 \gg \xi_\chi \chi^2$ and includes also the reheating stage.

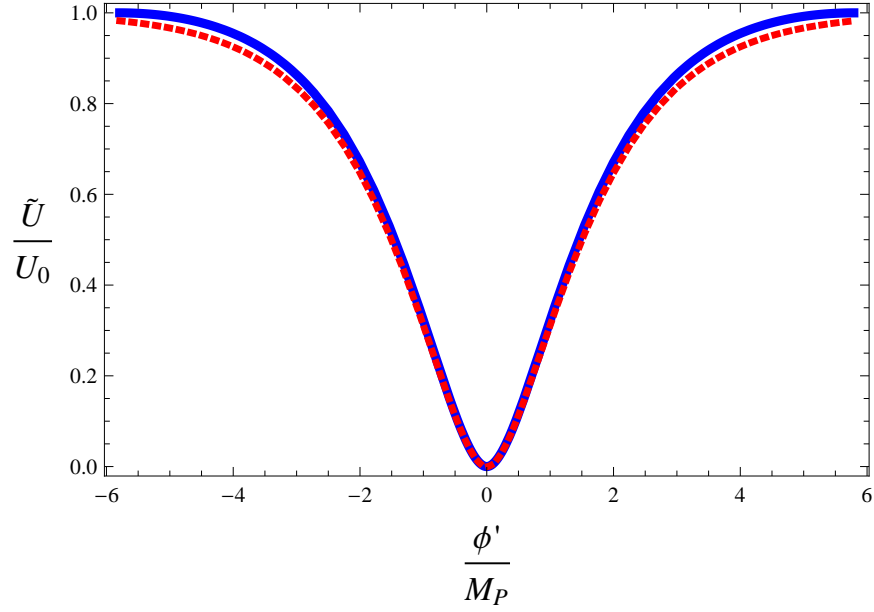


FIG. 1. Comparison between the Higgs-Dilaton inflationary potential (blue continuous line) obtained from (2.23) in the scale-invariant region and the corresponding one for the Higgs Inflation model (red dotted line). The amplitudes are normalized to the asymptotic value $U_0 = \frac{\lambda M_P^4}{4\xi_h^2}$.

where

$$\gamma = \sqrt{\frac{\xi_\chi}{1 + 6\xi_\chi}} \quad \text{and} \quad a = \sqrt{\frac{\xi_\chi(1 - \varsigma)}{\varsigma}}. \quad (2.21)$$

The variable ϕ' is periodic and defined in the compact interval $\phi' \in [-\phi_0, \phi_0]$, with $\phi_0 = M_P/a \tanh^{-1}[\sqrt{1 - \varsigma}]$ the value of the field at the beginning of inflation. In terms of these variables the Lagrangian (2.12) takes a very simple form⁷

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_P^2}{2} \tilde{R} - \frac{\varsigma \cosh^2[a\phi/M_P]}{2} (\partial r)^2 - \frac{1}{2} (\partial \phi)^2 - \tilde{U}(\phi) - \tilde{U}_{\Lambda_0}(r, \phi), \quad (2.22)$$

with $\phi = \phi_0 - |\phi'|$. The potential (2.19) becomes

$$\tilde{U}(\phi) = \frac{\lambda M_P^4}{4\xi_h^2(1 - \varsigma)^2} (1 - \varsigma \cosh^2[a\phi/M_P])^2, \quad \tilde{U}_{\Lambda_0}(r, \phi) = \frac{\Lambda_0}{\gamma^2} \varsigma^2 \cosh^4[a\phi/M_P] e^{-4\gamma r/M_P}, \quad (2.23)$$

whose scale-invariant part $\tilde{U}(\phi)$ resembles the potential of the simplest Higgs inflationary scenario [2], cf. Fig. 1. The analytical expressions for the amplitude and the spectral tilt of scalar perturbations at order $\mathcal{O}(\xi_\chi, 1/\xi_h, 1/N^*)$ can be easily calculated to obtain [27]

$$P_\zeta(k_0) \simeq \frac{\lambda \sinh^2[4\xi_\chi N^*]}{1152\pi^2 \xi_\chi^2 \xi_h^2}, \quad n_s(k_0) \simeq 1 - 8\xi_\chi \coth(4\xi_\chi N^*), \quad (2.24)$$

where N^* denotes the number of e-folds between the moment at which the pivot scale $k_0/a_0 = 0.002 \text{ Mpc}^{-1}$ exited the horizon and the end of inflation. Note that for $1 < 4\xi_\chi N^* \ll 4N^*$, the expression for the tilt simplifies and becomes linear in ξ_χ

$$n_s(k_0) \simeq 1 - 8\xi_\chi. \quad (2.25)$$

⁷ Note that the definition of the angular variable ϕ used in this work is slightly different from that appearing in Ref. [27]. The new parametrization makes explicit the symmetry of the potential and shifts its minimum to make it coincide with that in Higgs-inflation.

An interesting cosmological phenomenology arises with the peculiar choice⁸ $\beta = 0$. In this case, the DE dominated period in the late Universe depends only on the dilaton field ρ , which give rise to an intriguing relation between the inflationary and DE domination periods. Let us start by noticing that around the minimum of the potential the value of θ is very close to zero. In that limit, $\tan^2 \theta \ll \eta$, which prevents the use of the field redefinition (2.20). The appropriate redefinitions needed to diagonalize the kinetic term (2.17) in this case turn out to be

$$r = \gamma^{-1} \rho \quad \text{and} \quad \phi' \simeq \frac{M_P}{\sqrt{\xi_h \varsigma}} \theta. \quad (2.26)$$

Using Eqs. (2.17) and (2.19) it is straightforward to show that the part of the theory associated to the Higgs field ϕ simplifies to the SM one. The resulting scale-invariance breaking potential for the dilaton is still of the “run-away” type

$$\tilde{U}_{\Lambda_0}(r) = \frac{\Lambda_0}{\gamma^2} \varsigma^2 e^{-4\gamma r/M_P}, \quad (2.27)$$

making it suitable for playing the role of quintessence. Let us assume that \tilde{U}_{Λ_0} is negligible during the radiation and matter dominated stages but responsible for the present accelerated expansion of the Universe. In that case, it is possible to write the following relation between the equation of state parameter ω_r of the r field and its relative abundance Ω_r [36]

$$1 + \omega_r = \frac{16\gamma^2}{3} \left[\frac{1}{\sqrt{\Omega_r}} - \frac{1}{2} \left(\frac{1}{\Omega_r} - 1 \right) \log \frac{1 + \sqrt{\Omega_r}}{1 - \sqrt{\Omega_r}} \right]^2. \quad (2.28)$$

For the present DE density $\Omega_{\text{DE}} = \Omega_r \simeq 0.74$, the above expression yields

$$1 + \omega_{\text{DE}} = \frac{8}{3} \frac{\xi_\chi}{1 + 6\xi_\chi}. \quad (2.29)$$

Comparing Eqs. (2.25) and (2.29), it follows that the deviation of the scalar tilt n_s from the scale-invariant one is proportional to the deviation of the DE equation of state from a cosmological constant⁹ [27]

$$n_s - 1 \simeq -3(1 + \omega_{\text{DE}}), \quad \text{for} \quad \frac{2}{3N^*} < 1 + \omega_{\text{DE}} \ll 1. \quad (2.30)$$

The above condition is a non-trivial prediction of Higgs-Dilaton cosmology, relating two a priori completely independent periods in the history of the Universe. This has interesting consequences from an observational point of view¹⁰ and makes the Higgs-Dilaton scenario rather unique. We will be back to this point in Section IV, where we will show that the consistency relation (2.30) still holds even in the presence of quantum corrections computed within the “minimal setup”.

III. THE DYNAMICAL CUT-OFF SCALE

Following Ref. [24], we now turn to the determination of the energy domain where the Higgs-Dilaton model can be considered as a predictive effective field theory. This domain is bounded from above by the field-dependent cut-off $\Lambda(\Phi)$, i.e. the energy where perturbative tree-level unitarity is violated [37]. At energies above that scale, the theory becomes strongly-coupled and the standard perturbative methods fail. In order to determine this (background dependent) energy scale, two related methods, listed below, can be used.

1. Expand the generic fields of the theory around their background values

$$\Phi(\mathbf{x}, t) = \bar{\Phi} + \delta\Phi(\mathbf{x}, t), \quad (3.1)$$

⁸ Some arguments in favour of the $\beta = 0$ case can be found in Ref. [27, 30, 35].

⁹ Outside this region of parameter space, the relation connecting n_s to ω_{DE} is somehow more complicated

$$n_s - 1 \simeq -\frac{12(1 + \omega_{\text{DE}})}{4 - 9(1 + \omega_{\text{DE}})} \coth \left[\frac{6N^*(1 + \omega_{\text{DE}})}{4 - 9(1 + \omega_{\text{DE}})} \right].$$

¹⁰ Similar consistency relations relating the rate of change of the equation of state parameter $w(a) = w_0 + w_a(1 - a)$ with the logarithmic running of the scalar tilt can be also derived, cf. Ref. [27]. The practical relevance of those consistence conditions is however much more limited than that of Eq. (2.30), given the small value of the running of the scalar tilt in Higgs-driven scenarios.

such that all kind of higher-dimensional non-renormalizable operators

$$c_n \frac{\mathcal{O}_n(\delta\Phi)}{[\Lambda(\bar{\Phi})]^{n-4}} , \quad (3.2)$$

with $c_n \sim \mathcal{O}(1)$ appear in the resulting action. These operators are suppressed by appropriate powers of the field-dependent coefficient $\Lambda(\bar{\Phi})$, which can be identified as the cut-off of the theory. This procedure gives us only a lower estimate of the cut-off, since it does not take into account the possible cancelations that might occur between the different scattering diagrams.

2. Calculate at which energy each of the N-particle scattering amplitudes hit the unitarity bound. The cut-off will then be the lowest of these scales.

In what follows we will apply these two methods to determine the effective cut-off of the theory. We will start by applying the method 1) to compute the cut-off associated with the gravitational and scalar interactions. The cut-off associated to the gauge and fermionic sectors will be obtained via the method 2).

A. Cut-off in the scalar-gravity sector

We choose to work in the original Jordan frame where the Higgs and dilaton fields are non-minimally coupled to gravity. Expanding these fields around a static background¹¹

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} , \quad \chi = \bar{\chi} + \delta\chi , \quad h = \bar{h} + \delta h , \quad (3.3)$$

we obtain the following kinetic term for the quadratic Lagrangian of the gravity and scalar sectors

$$\begin{aligned} \mathcal{K}_2^{\text{G+S}} = & \frac{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}{8} (\delta g^{\mu\nu} \square \delta g_{\mu\nu} + 2 \partial_\nu \delta g^{\mu\nu} \partial^\rho \delta g_{\mu\rho} - 2 \partial_\nu \delta g^{\mu\nu} \partial_\mu \delta g - \delta g \square \delta g) \\ & - \frac{1}{2} (\partial \delta \chi)^2 - \frac{1}{2} (\partial \delta h)^2 + (\xi_\chi \bar{\chi} \delta \chi + \xi_h \bar{h} \delta h) (\partial_\lambda \partial_\rho \delta g^{\lambda\rho} - \square \delta g) . \end{aligned} \quad (3.4)$$

The leading higher-order non-renormalizable operators obtained in this way are given by

$$\xi_\chi (\delta \chi)^2 \square \delta g , \quad \xi_h (\delta h)^2 \square \delta g . \quad (3.5)$$

Note that these operators are written in terms of quantum excitations with non-diagonal kinetic terms. In order to properly identify the cut-off of the theory, we should determine the normal modes that diagonalize the quadratic Lagrangian (3.4). After doing that, and using the equations of motion to eliminate artificial degrees of freedom, we find that the metric perturbations depend on the scalar fields perturbations, a fact that is implicit in the Lagrangian (3.4). The gravitational part of the above action can be recast into canonical form in terms of a new metric perturbation $\delta \hat{g}_{\mu\nu}$ given by

$$\delta \hat{g}_{\mu\nu} = \frac{1}{\sqrt{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}} [(\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2) \delta g_{\mu\nu} + 2 \bar{g}_{\mu\nu} (\xi_\chi \bar{\chi} \delta \chi + \xi_h \bar{h} \delta h)] . \quad (3.6)$$

The cut-off scale associated to purely gravitational interactions becomes in this way the effective Planck scale in the Jordan frame

$$\Lambda_P^2 = \xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2 . \quad (3.7)$$

The remaining non-diagonal kinetic term for the scalar perturbations $(\delta \Phi^1, \delta \Phi^2) = (\delta \chi, \delta h)$ is given in compact matrix notation by

$$\mathcal{K}_2^{\text{S}} = -\frac{1}{2} \bar{\kappa}_{ij}^J \partial_\mu \delta \Phi^i \partial^\mu \delta \Phi^j , \quad (3.8)$$

¹¹ Note that, in comparison with the analysis performed in Ref. [20] for generalized Higgs inflationary models, both the dilaton and the Higgs field acquire a non-zero background expectation value, cf. Section II. As we will see below, this will give rise to a much richer cutoff structure.

where $\bar{\kappa}_{ij}^J = \Omega^2 \bar{\kappa}_{ij}^E$ is the Jordan frame analogue of Eq. (2.15) and depends only on the background values of the fields, i.e.

$$\bar{\kappa}_{ij}^J = \frac{1}{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2} \begin{pmatrix} \xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 & 6\xi_\chi \bar{\chi} \xi_h \bar{h} \\ 6\xi_\chi \bar{\chi} \xi_h \bar{h} & \xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2 (1 + 6\xi_h) \end{pmatrix}. \quad (3.9)$$

In order to diagonalize the above expression we make use of the following set of variables

$$\begin{aligned} \delta\hat{\chi} &= \sqrt{\frac{\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h)}{(\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2)(\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2)}} (\xi_\chi \bar{\chi} \delta\chi + \xi_h \bar{h} \delta h), \\ \delta\hat{h} &= \frac{1}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2}} (-\xi_h \bar{h} \delta\chi + \xi_\chi \bar{\chi} \delta h). \end{aligned} \quad (3.10)$$

Note here that this is precisely the change of variables (up to an appropriate rescaling with the conformal factor Ω) needed to diagonalize the kinetic terms for the scalar perturbations in the Einstein frame. To see this, it is enough to start from Eq. (2.14) and expand the fields around their background values $\Phi^i \rightarrow \bar{\Phi}^i + \delta\Phi^i$. Keeping the terms with the lowest power in the excitations, $\tilde{K} = \bar{\kappa}_{ij}^E \partial_\mu \delta\Phi^i \partial^\mu \delta\Phi^j + \mathcal{O}(\delta\Phi^3)$, it is straightforward to show that the previous expression can be diagonalized in terms of

$$\begin{aligned} \delta\hat{\chi} &= \bar{\Omega}^{-1} \sqrt{\frac{\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h)}{(\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2)(\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2)}} (\xi_\chi \bar{\chi} \delta\chi + \xi_h \bar{h} \delta h), \\ \delta\hat{h} &= \bar{\Omega}^{-1} \frac{1}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2}} (-\xi_h \bar{h} \delta\chi + \xi_\chi \bar{\chi} \delta h). \end{aligned} \quad (3.11)$$

Written in terms of the canonically normalized variables (3.6) and (3.10) these operators read

$$\frac{1}{\Lambda_1} (\delta\hat{h})^2 \square \delta\hat{g}, \quad \frac{1}{\Lambda_2} (\delta\hat{\chi})^2 \square \delta\hat{g}, \quad \frac{1}{\Lambda_3} (\delta\hat{\chi})(\delta\hat{h}) \square \delta\hat{g}, \quad (3.12)$$

where the different cut-off scales are given by

$$\Lambda_1 = \frac{\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2}{\xi_\chi \xi_h \sqrt{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}}, \quad (3.13)$$

$$\Lambda_2 = \frac{(\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2)(\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h))}{(\xi_\chi^3 \bar{\chi}^2 + \xi_h^3 \bar{h}^2) \sqrt{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}}, \quad (3.14)$$

$$\Lambda_3 = \frac{(\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2)(\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h))}{\xi_\chi \bar{\chi} \xi_h \bar{h} |\xi_h - \xi_\chi| \sqrt{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}}. \quad (3.15)$$

The effective cut-off of the scalar theory at a given value of the background fields will be the lowest of the previous scales. We will be back to this point in Section III C.

B. Cut-off in the gauge and fermionic sectors

Let us now move to the cut-off associated with the gauge sector. Since we are working in the unitary gauge for the SM fields, it is sufficient to look at the tree-level scattering of non-abelian vector fields with longitudinal polarization. It is well known that in the SM the “good” high energy behaviour of these processes is the result of cancellations that occur when we take into account the interactions of the gauge bosons with the excitations δh of the Higgs field¹² [38, 39].

¹² In the absence of the Higgs field, the scattering amplitudes grow as the square of the center-of-mass energy, due to the momenta dependence of the longitudinal vectors $\sim q^\mu/m_W$.

In our case, even though purely gauge interactions remain unchanged, the graphs involving the Higgs field excitations are modified due to the non-canonical kinetic term. This changes the pattern of the cancellations that occur in the standard Higgs mechanism, altering therefore the asymptotic behaviour of these processes. As a result, the energy scale where this part of the theory becomes strongly coupled becomes lower.

To illustrate how this happens, let us consider the $W_L W_L \rightarrow W_L W_L$ scattering in the s -channel. The relevant part of the Lagrangian is

$$g m_W W_\mu^+ W^{-\mu} \delta h, \quad (3.16)$$

where $m_W \sim g \bar{h}$. After diagonalizing the kinetic term for the scalar fields with the change of variables (3.10), the above expression becomes

$$g' m_W W_\mu^+ W^{-\mu} \delta \hat{h} + g'' m_W W_\mu^+ W^{-\mu} \delta \hat{\chi}, \quad (3.17)$$

where the effective coupling constants g' and g'' are given by

$$g' = g \frac{\xi_\chi \bar{\chi}}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h \bar{h}^2}}, \quad g'' = g \frac{\xi_h \bar{h}}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h \bar{h}^2}} \sqrt{\frac{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}{\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h)}}. \quad (3.18)$$

From the requirement of tree unitarity of the S -matrix, it is straightforward to show that the scattering amplitude of this interaction hits the perturbative unitarity bound at energies

$$\Lambda_G \simeq \sqrt{\frac{\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h)}{6\xi_h^2}}. \quad (3.19)$$

It is interesting to compare the previous expression with the results for the gauge cutoff of the simplest Higgs inflationary model [24]. In order to do that, let us consider two limiting cases: the inflationary/high-energy period corresponding to field values $\xi_\chi \bar{\chi}^2 \ll \xi_h \bar{h}^2$ and the low-energy regime at which $\xi_\chi \bar{\chi}^2 \gg \xi_h \bar{h}^2$. In these two cases, the above expression simplifies to

$$\Lambda_G \simeq \begin{cases} \bar{h} & \text{for } \xi_\chi \bar{\chi}^2 \ll \xi_h \bar{h}^2, \\ \frac{\sqrt{\xi_\chi \bar{\chi}}}{\xi_h} & \text{for } \xi_\chi \bar{\chi}^2 \gg \xi_h \bar{h}^2, \end{cases} \quad (3.20)$$

in agreement with the Higgs inflation model.

To identify the cut-off of the fermionic part of the Higgs-Dilaton model, we consider the chirality non-conserving process $\bar{f} f \rightarrow W_L W_L$. This interaction receives contributions from diagrams with γ and Z exchange (s -channel) and from a diagram with fermion exchange (t -channel). In the asymptotic high-energy limit, the total amplitude of these graphs grows linearly with the energy at the center of mass. Once again, the s -channel diagram including the Higgs excitations unitarizes the associated amplitude [40–42]. Following therefore the same steps as in the calculation of the gauge cut-off, we find that this part of the theory enters into the strong-coupling regime at energies

$$\Lambda_F \simeq y^{-1} \frac{\xi_\chi \bar{\chi}^2 (1 + 6\xi_\chi) + \xi_h \bar{h}^2 (1 + 6\xi_h)}{6\xi_h^2 \bar{h}}, \quad (3.21)$$

where y is the Yukawa coupling constant. The above cut-off is higher than that of the SM gauge interactions (3.19) during the whole evolution of the Universe.

C. Comparison with the energy scales in the early and late Universe

In this section we compare the cut-offs found above with the characteristic energy scales in the different periods during the evolution of the Universe. If the typical momenta involved in the different processes are sufficiently small, the theory will remain in the weak coupling limit, making the Higgs-Dilaton scenario self-consistent.

Let us start by considering the inflationary period, characterized by $\xi_h \bar{h}^2 \gg \xi_\chi \bar{\chi}^2$. As shown in Fig. 2, the lowest cut-off in this region is the one associated with the gauge interactions Λ_G . The typical momenta of the scalar perturbations produced during inflation are of the order of the Hubble parameter at that time. This quantity can be easily estimated in the Einstein frame, where it is basically determined by the energy stored in the inflationary potential (2.23). We obtain $\bar{H} \sim \sqrt{\lambda} M_P / \xi_h$. When transformed to the Jordan frame ($H = \Omega \bar{H}$) this quantity becomes

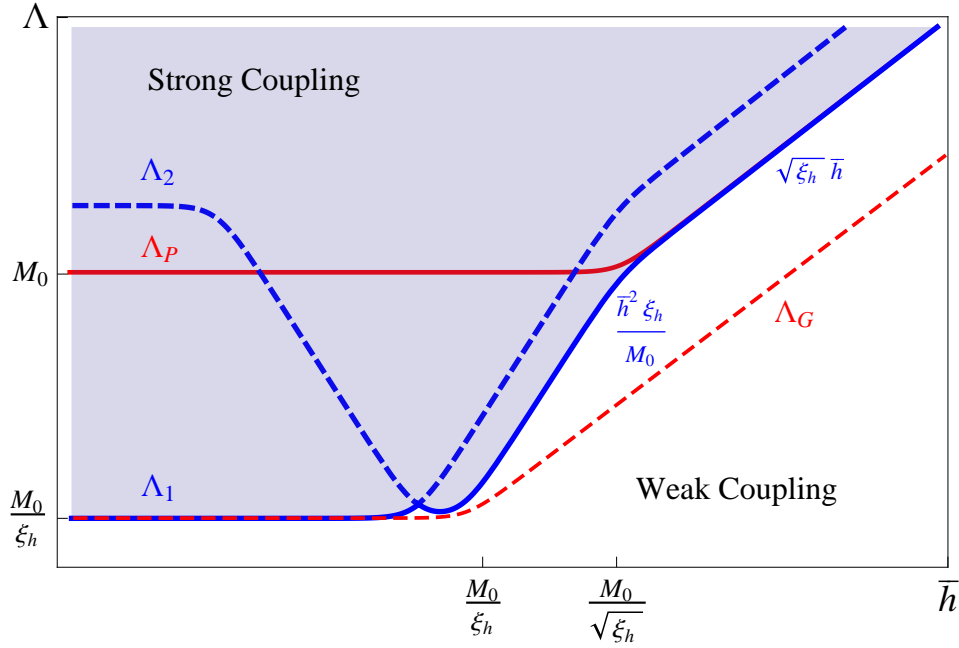


FIG. 2. Dependence of the different cut-off scales for a fixed value of the dilaton field $\bar{\chi}$ as a function of the Higgs field \bar{h} in the Jordan frame. The cut-off (3.15) is parametrically above the other energy scales (Λ_1 , Λ_2 , Λ_P , Λ_G and Λ_F) during the whole history and it is therefore not included in the figure. The effective field theory description of scalar fields is applicable for typical energies below the thick blue solid line, which correspond to the minimum of the scalar cut-off scales at a given field value. This is given by Λ_2 and Λ_1 in the scalar sector, for large and small Higgs values respectively. The red solid line correspond to the gravitational cut-off (3.7), while the red dashed one corresponds to the gauge cut-off (3.19). They coincide with the effective scalar sector cut-off for large and small Higgs values respectively. The scale M_0 is defined as $M_0 = \sqrt{\xi_h} \bar{\chi}$ and corresponds to the value of the effective Planck mass at low energies.

$H \sim \sqrt{\frac{\lambda}{\xi_h}} \bar{h}$, which is significantly below the cut-off scale Λ_G in that region. The same conclusion is obtained for the total energy density, which turns out to be much smaller than Λ_G^4 . Moreover, the cut-off Λ_G exceeds the masses of all particles in the Higgs background, allowing a self-consistent estimate of radiative corrections (cf. Section IV).

After the end of inflation, the field ϕ starts to oscillate around the minimum of the potential with a decreasing amplitude, due to the expansion of the Universe and particle production. This amplitude varies between $M_0/\sqrt{\xi_h}$ and M_0/ξ_h , where $M_0 = \sqrt{\xi_h} \bar{\chi}$ is the asymptotic Planck scale in the low energy regime. As shown in Fig. 1, the curvature of the Higgs-Dilaton potential around the minimum coincides (up to $\mathcal{O}(\xi_h)$ corrections) with that of the Higgs-inflation scenario. All the relevant physical scales, including the effective gauge and fermion masses, agree, up to small corrections, with those in Higgs-inflation [43]. This allows us to directly apply the results of [3, 4, 44] to the Higgs-Dilaton scenario. According to these works, the typical momenta of the gauge bosons produced at the minimum of the potential in the Einstein frame is of order $\tilde{k} \sim (\tilde{m}_A/M)^{2/3} M$, with \tilde{m}_A the mass of the gauge bosons in the Einstein frame and $M = \sqrt{\lambda/3} M_P/\xi_h$ the curvature of the potential around the minimum. After transforming to the Jordan frame we obtain $k \sim \left(\frac{\lambda g^4}{\xi_h}\right)^{1/6} \Lambda_G$, with g the weak coupling constant. The typical momentum of the created gauge bosons is therefore parametrically below the gauge cut-off scale (3.20) in that region.

At the end of the reheating period, $\xi_h \bar{\chi}^2 \gg \xi_h \bar{h}^2$, the system settles down to the minimum of the potential $\tilde{U}(\phi)$, cf. Eq. (2.23). In that region the effective Planck mass coincides with the value M_0 . The cut-off scale becomes $\Lambda_1 \simeq \sqrt{\xi_h} \bar{\chi}/\xi_h \simeq M_P/\xi_h$. This value is much higher than the electroweak scale $m_H^2 \sim 2\alpha/\xi_h M_P$ (cf. Eq. (2.5)) where all the physical processes take place. We conclude therefore that perturbative unitarity is maintained for all the relevant processes during the whole evolution of the Universe.

IV. QUANTUM CORRECTIONS

In this section we concentrate on the radiative corrections to the inflationary potential and on their influence on the predictions of the model.

Our strategy is as follows. We regularize the quantum theory in such a way that all multi-loop diagrams are finite, whereas the exact symmetries of the classical action (gauge, diffeomorphisms and scale invariance) remain intact. Moreover, we will require the regularization to respect the approximate shift symmetry of the dilaton field in the Jordan frame, cf. Section II. Then we add to the classical action (2.9) an infinite number of counter-terms (including the finite parts as well) which remove all the divergences from the theory and do not spoil the exact and approximate symmetries of the classical action. Since the theory is not renormalizable, these counter-terms will have a different structure from that of the classical action. In particular, terms that are non-analytic with respect to the Higgs and dilaton fields will appear [45]. They can be considered as higher-dimensional operators, suppressed by the field-dependent cut-offs. For consistency with the analysis made earlier in this work, we demand these cut-offs to exceed those found in Section III.

An example of the subtraction procedure which satisfies all the requirements formulated above has been constructed in Ref. [35] (see also earlier discussion in [46]). It is based on dimensional regularization in which the 't Hooft-Veltman normalization point μ is replaced by some combination of the scalar fields with an appropriate dimension, $\mu^2 \rightarrow F(\chi, h)$ (we underline that we use the Jordan frame here for all definitions). The infinite part of the counter-terms is defined as in \overline{MS} prescription, i.e. by subtracting the pole terms in ϵ , where the dimensionality of space-time is $D = 4 - 2\epsilon$. The finite part of the counter-terms has the same operator structure as the infinite part, including the parametric dependence on the coupling constants. The requirement of the structure of higher-dimensional operators, formulated in the previous paragraph, puts an important constraint on the choice of the function $F(\chi, h)$, as it is this combination that appears in the denominator of the counter-terms [35, 45]. The simplest and most natural choice is to identify the normalization point in the Jordan frame with the gravitational cut-off (3.7), $\mu_I^2 \propto \xi_\chi \chi^2 + \xi_h h^2$, which corresponds to the scale-invariant prescription of Ref. [35]. In the Einstein frame the previous choice becomes standard (field-independent)

$$\tilde{\mu}_I^2 \propto M_P^2. \quad (4.1)$$

A second possibility is to choose the scale-invariant direction along the dilaton field, i.e. $\mu_{II}^2 \propto \xi_\chi \chi^2$. When transformed to the Einstein frame it becomes

$$\tilde{\mu}_{II}^2 \propto \frac{\xi_\chi \chi^2 M_P^2}{\xi_\chi \chi^2 + \xi_h h^2}, \quad (4.2)$$

which coincide with the prescription II of Ref. [10] at the end of inflation.

In what follows we will use this “minimal setup” for the analysis of the radiative corrections. It will be more convenient to work in the Einstein frame, where the coupling to gravity is minimal and all non-linearities are moved to the matter sector. The total action in the Einstein frame naturally divides into an Einstein-Hilbert part (EH), a purely scalar piece involving only the Higgs and dilaton fields (HD) and a part corresponding to the chiral SM without the radial mode of the Higgs boson (CH) [10, 47, 48]

$$S = S_{\text{EH}} + S_{\text{HD}} + S_{\text{CH}}. \quad (4.3)$$

In the next section we estimate the contribution of the scalar sector to the effective inflationary potential, postponing the study of the chiral SM to Section IV B. All the computations will be performed in flat spacetime, since the inclusion of gravity does not modify the results¹³.

A. Scalar contribution to the effective inflationary potential

Let us start by reminding that the initial value of the dilaton field has to be sufficiently large to keep its present contribution to DE at the appropriate observational level [27]. The latter fact allows us to neglect the exponentially suppressed contributions to the effective action stemming from \tilde{U}_{Λ_0} in Eq. (2.23). As a result, the remaining corrections due to the dilaton field will emerge from its non-canonical kinetic term, whereas all the radiative corrections due to the Higgs field will emerge from the inflationary potential.

The construction of the effective action for the scalar sector of the theory is most easily done in the following way: expand the action (2.22) near the constant background of the dilaton and the Higgs fields and drop the linear terms in perturbations. After that, compute all the vacuum diagrams to account for the potential-type corrections and all the diagrams with external legs to account for the kinetic-type corrections.

¹³ We recall that, in the Einstein frame, the coupling among SM particles and gravity is minimal.



FIG. 3. Some of the two-loop diagrams for the dilaton.

2. Higgs contribution

We now turn to the corrections to the Higgs field. Once again we consider first the potential-type contributions. The situation now is more complicated, since the effective potential for the Higgs field ϕ will be modified by terms stemming from the scale-invariant part of the tree-level potential (2.23) as well as from the non-canonical kinetic term of the dilaton field r , with the latter starting from the second order in perturbation theory.

Let us start by considering the contributions due to the tree-level potential. To keep the notation as simple as possible, we express the scale-invariant part of the potential (2.23) in the following compact form

$$\tilde{U}(\phi) = U_0 \left(u_0 + \sum_{n=1}^2 u_n \cosh[2na\phi/M_P] \right), \quad U_0 = \frac{M_P^4}{4\xi_h^2(1-\varsigma)^2}, \quad (4.6)$$

where, for completion, we have explicitly recovered the α and β dependence and defined

$$u_0 = c^2 - c\sigma + \frac{3\sigma^2}{8} + \frac{3\beta'}{2}, \quad u_1 = \frac{\sigma^2}{2} - c\sigma - 2\beta', \quad u_2 = \frac{\sigma^2}{8} + \frac{\beta'}{2}, \quad (4.7)$$

with

$$c = 1 + \frac{\alpha}{\lambda} \frac{1 + 6\xi_h}{1 + 6\xi_\chi}, \quad \sigma = \varsigma + \frac{\alpha}{\lambda} \frac{1 + 6\xi_h}{1 + 6\xi_\chi}, \quad \beta' \equiv \frac{\beta}{\lambda} \left(\frac{1 + 6\xi_h}{1 + 6\xi_\chi} \right)^2. \quad (4.8)$$

Expanding the field around its background value $\bar{\phi}$, we get

$$\begin{aligned} \lambda \tilde{U}(\bar{\phi} + \delta\phi) &= \lambda U_0 \sum_{n=1}^2 u_n \sum_{l=0}^{\infty} \frac{\cosh^{(l)}[2na\bar{\phi}/M_P]}{l!} \left(\frac{2na\delta\phi}{M_P} \right)^l \\ &= \lambda U_0 \sum_{n=1}^2 \sum_{l=0}^{\infty} u_n \left[c_{n,l} \cosh[2na\bar{\phi}/M_P] \left(\frac{a\delta\phi}{M_P} \right)^{2l} + d_{n,l} \sinh[2na\bar{\phi}/M_P] \left(\frac{a\delta\phi}{M_P} \right)^{2l+1} \right], \end{aligned} \quad (4.9)$$

where $c_{n,l}$ and $d_{n,l}$ account for numerical coefficients and combinatorial factors. Since the theory is non-renormalizable, the perturbative expansion creates terms which do not have the same background dependence of the original potential. Up to numerical factors, the contributions turn out to be of the form¹⁶

$$\frac{\lambda^{i+j} M_P^4}{[4\xi_h^2(1-\varsigma)^2]^{i+j}} \left[g\left(\frac{1}{\epsilon}\right) + f_{i,j} \right] \sum_{n,m} u_n^i u_m^j \cosh^i[2na\bar{\phi}/M_P] \sinh^j[2ma\bar{\phi}/M_P], \quad (4.10)$$

where $f_{i,j}$ denotes the (finite) integration constant, and $g(1/\epsilon)$ is a function of the divergent terms. Note that if we set $\beta = 0$, we make sure that terms which contribute to the cosmological constant (2.6) will not be generated by the loop expansion.

By inspection of the structure of divergences, we can see that the leading corrections are those appearing with the lowest power in ς . To gain insight on their contribution, we calculate the finite part of Eq. (4.10) for the maximal value of the hyperbolic functions. This corresponds to $\phi_{\max} = \phi_0 \equiv M_P/\alpha \tanh^{-1}[\sqrt{1-\varsigma}]$. We get

$$\frac{\lambda^{i+j}}{[4\xi_h^2(1-\varsigma)^2]^{i+j}} f_{i,j} \sum_{n,m} u_n^i u_m^j \cosh^i[2na\bar{\phi}/M_P] \sinh^j[2ma\bar{\phi}/M_P] \Big|_{\bar{\phi}=\phi_{\max}} \sim \left(\frac{\lambda\varsigma}{4\xi_h^2} \right)^{i+j} f_{i,j}, \quad (4.11)$$

¹⁶ To maintain the expressions as compact as possible we decided not to express the result in terms of m_H/M_P .

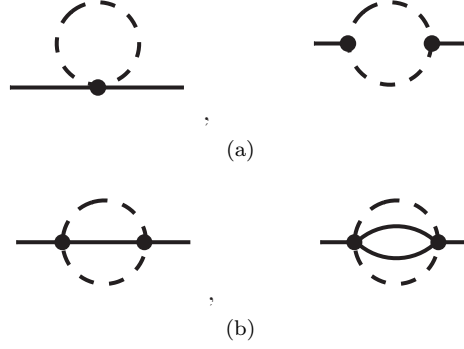


FIG. 4. Characteristic diagrams produced by the non-canonical kinetic term of the dilaton field r . Solid and dashed lines represent the Higgs and dilaton fields respectively. The first one-loop diagram presented in (a) vanishes in dimensional regularization due to the massless character of the dilaton field. On the other hand, the second diagram gives rise to higher derivative terms of the Higgs field. In (b) we consider two and three loop diagrams which, apart from generating higher dimensional operators, contribute to the effective potential once we amputate them.

which makes the corrections coming from the order $i + j + 1$ negligible compared to the ones from $i + j$ order. In the last step we have simply set $c = 1$, $\sigma = \varsigma$, which, given the small value of the parameter α appearing in Eq. (4.8), constitutes a very good approximation.

As we mentioned earlier, potential-type corrections to the Higgs field are also generated from diagrams associated to the kinetic term of the dilaton r , starting from two loops. This happens because the first order vacuum diagrams with dilaton running in the loop, vanish. If we consider higher loop diagrams, like those in Fig. 4(b) but without momenta in the external legs, we see that even though the background dependence of the corrections is complicated due to the non-canonically normalized dilaton that runs inside the loops, their contributions to the effective action are of the same order as those in Eq. (4.11).

We now turn to the kinetic-type corrections to the Higgs field. By that we mean corrections to the propagator, as well as terms with more derivatives of the field suppressed by the scalar cut-off. The first type of contributions come only from the scale-invariant part of the potential given by (4.6), when the momenta associated to the external legs are considered. It is not difficult to show that these are precisely of the same form as those in (4.10). The second type of contributions, i.e. the higher dimensional operators, are generated both from the Higgs potential at higher loops, as well as from the non-vanishing diagrams associated to the non-canonical kinetic term of the dilaton. The terms we get are proportional to

$$\frac{\partial^2}{M_P^2}(\partial\phi)^2, \quad \frac{\partial^4}{M_P^4}(\partial\phi)^2 \dots, \quad (4.12)$$

and they can be safely neglected for the typical momenta involved in the different epochs of the evolution of the Universe.

Before moving on, we would like to comment on the appearance of mixing terms with derivatives of the fields. These manifest themselves when we consider diagrams with both fields in the external legs. They are higher dimensional operators, and it can be shown that they appear suppressed by the scalar cut-off of the theory, as before.

Since the kinetic-type operators do not modify the dynamics, we will consider only potential-type corrections to estimate the change in the tree-level predictions of the model. At one-loop, the contribution of the scalar sector to the inflationary potential becomes [50]

$$\Delta\tilde{U}_{HD} \simeq \frac{U_0}{64\pi^2} \frac{\lambda a^4}{\xi_h^2(1-\varsigma)^2} \left(\frac{1}{\bar{\epsilon}} + f_{2,0} \right) \left[\varsigma^2 \frac{1 + \cosh[4a\bar{\phi}/M_P]}{2} + \mathcal{O}(\varsigma^3) \right], \quad (4.13)$$

where we just kept the leading contribution in ς . The finite part $f_{2,0}$ in the previous expression is given by

$$f_{2,0} = \frac{3}{2} - \log \left[\frac{-\tilde{U}''(\bar{\phi})}{\mu^2} \right] = \frac{3}{2} - \log \left[\frac{\lambda a^2 M_P^2}{\xi_h^2(1-\varsigma)^2 \mu^2} \left(\varsigma \cosh[2a\bar{\phi}/M_P] + \mathcal{O}(\varsigma^2) \right) \right]. \quad (4.14)$$

If we adopt the \overline{MS} scheme, the remaining (logarithmic) corrections will be suppressed by an overall factor $\mathcal{O}(10^{-15})$ (apart from different powers of ς) with respect to the tree-level potential (4.6). The quantum contribution of the

scalar sector to the effective inflationary potential is therefore completely negligible and rather insensitive to the particular choice of the renormalization point μ . This allows us to approximate the value of ϕ at the end of inflation by its classical value $\phi_f \simeq M_P/a \tanh^{-1}[\sqrt{1-\varsigma} \cos(2 \times 3^{1/4} \sqrt{\xi_\chi})]$, and compute analytically the spectral tilt n_s of primordial scalar perturbations, which turns out to be

$$n_s(k_0) - 1 \simeq -8\xi_\chi + \frac{\lambda\xi_\chi^2}{96\pi^2\xi_h^2} f_{2,0}, \quad \text{for } 1 \lesssim 4\xi_\chi N^* \ll 4N^*. \quad (4.15)$$

We see therefore that the correction to the tree-level result is controlled by the effective self-coupling of the Higgs field in the Einstein frame λ/ξ_h^2 . The small value of this parameter makes the scalar radiative contribution completely negligible and thus hardly modify the consistency relation (2.30). Note however that there might be still a significant contribution to the inflationary potential coming from the SM particles, especially from those with a large coupling to the Higgs field. The study of this effect is the purpose of the next section.

B. Chiral SM contribution to the effective inflationary potential.

The action for the SM fields during the inflationary stage is similar to that appearing in Higgs inflation [10] and takes the form of a chiral SM with a nearly decoupled Higgs field. Its contribution to the effective potential can be analyzed by the methods presented in Ref. [10]. The one-loop contribution during inflation reads¹⁷

$$\Delta U_1 = \frac{6m_W^4}{64\pi^2} \left(\log \frac{m_W^2}{\mu^2} - \frac{5}{6} \right) + \frac{3m_Z^4}{64\pi^2} \left(\log \frac{m_Z^2}{\mu^2} - \frac{5}{6} \right) - \frac{3m_t^4}{16\pi^2} \left(\log \frac{m_t^2}{\mu^2} - \frac{3}{2} \right) \quad (4.16)$$

where m_W^2 , m_Z^2 and m_t^2 stand for the effective W, Z and top quark masses *in the frame where the computation is performed*. Whatever this frame is chosen to be, let us transform it into the Einstein-frame, where the inflationary observables can be computed using the standard techniques. The conformal transformation $\Delta\tilde{U}_1 = \Delta U_1/\Omega^4$ acts only on the coefficients of the logarithmic terms in (4.16), leaving their arguments completely unchanged. We obtain therefore

$$\Delta\tilde{U}_1 = \frac{6\tilde{m}_W^4}{64\pi^2} \left(\log \frac{m_W^2}{\mu^2} - \frac{5}{6} \right) + \frac{3\tilde{m}_Z^4}{64\pi^2} \left(\log \frac{m_Z^2}{\mu^2} - \frac{5}{6} \right) - \frac{3\tilde{m}_t^4}{16\pi^2} \left(\log \frac{m_t^2}{\mu^2} - \frac{3}{2} \right) \quad (4.17)$$

where the Einstein-frame masses \tilde{m}^2 are proportional to the effective vacuum expectation value of the Higgs field in the Einstein frame¹⁸

$$v^2(\phi) \equiv \frac{h^2}{\Omega^2} = \frac{M_P^2}{\xi_h(1-\varsigma)} \left(1 - \varsigma \cosh^2 \frac{a\phi}{M_P} \right), \quad (4.18)$$

which is a slowly varying function during inflation. This fact allow us to completely factor out the ϕ dependence in front of the logarithms in Eq. (4.17) and perform the analysis below as if v was a constant, $v \simeq M_P/\sqrt{\xi_h}$.

Note that the explicit dependence on the 't Hooft-Veltman normalization point μ in Eq. (4.16) is specious and will be immediately compensated by the running of the coupling constants. The renormalization procedure effectively replaces the tree level potential by

$$\tilde{U}_{\text{tree}}(\phi) = \frac{\lambda(\mu)}{4} v^4(\mu). \quad (4.19)$$

The induced μ dependence of the effective Higgs vacuum expectation value in the previous expression is known from the analysis of the chiral SM [10, 47]

$$16\pi^2\mu \frac{\partial}{\partial\mu} v^2 = \left(\frac{3}{2}g'^2 + 3g^2 - 6y_t^2 \right) v^2, \quad (4.20)$$

and can be transferred into the RG running of the non-minimal coupling of the Higgs field to gravity¹⁹ $\xi_h(\mu)$. A similar RG equation for $\lambda(\mu)$ can be obtained by requiring the full inflationary potential to be μ independent (for

¹⁷ We neglect the contribution (4.13) associated to the scalar sector, which, as shown in the previous section, turns out to be very small.

¹⁸ In particular we have $\tilde{m}_W^2(\phi) = \tilde{m}_Z^2(\phi) \cos^2 \theta_w = g^2/2 \cdot v^2(\phi)$ and $\tilde{m}_t^2(\phi) = y_t^2/2 \cdot v^2(\phi)$.

¹⁹ We safely neglect the running of the non-minimal coupling ξ_χ , which, given its tiny value and the fact that the dilaton is only weakly coupled to matter, is expected to be very small.

details cf. Ref. [10]). Once the RG running of the couplings is known, one can choose the value of μ in such a way that the logarithmic contribution, for each given value ϕ of the Higgs field, is minimized. In that case, the RG enhanced (RGE) inflationary potential becomes

$$\tilde{U}_{\text{RGE}}(\phi) = \frac{\lambda(\mu(\phi))}{4} \frac{M_P^4}{\xi_h^2(\mu(\phi))(1-\varsigma)^2} \left(1 - \varsigma \cosh^2 \frac{a\phi}{M_P}\right)^2, \quad (4.21)$$

which in fact suffices for any practical purposes, without the need of the 1-loop effective potential.

Note that the choice of the optimal value of μ is however a non-trivial task. All the relevant information about the initial frame in which the theory was quantized is encoded in the logarithmic terms in Eq. (4.17). Different conformally related frames give rise to different particle masses, which leads to an ambiguity in the choice of the normalization point μ needed to minimize the logarithms in the higher-loop corrections. This is a fundamental problem in the quantization of any scalar-tensor theory that cannot be solved without the knowledge of the UV completion at the Planck scale. In the language of the scale-invariant subtraction procedure described in Ref. [35] (cf. also the beginning of Section IV) this ambiguity is encoded in the particular choice of the function $F(\chi, h)$, which replaces the normalization point μ in Eq. (4.17). In what follows we will consider two “natural” choices, which closely correspond to those used in Refs. [7, 10]. Let us formulate them in the Jordan frame. The first choice is the “scale invariant” prescription

$$\mu_I^2 = \frac{\hat{\mu}^2}{M_P^2} (\xi_\chi \chi^2 + \xi_h h^2), \quad (4.22)$$

where the parameter $\hat{\mu}$ enables the use of RG methods for the improvement of the calculations at arbitrary energies²⁰. The RG enhancement of the potential in this case dictates

$$\hat{\mu}_I^2(\phi) = \frac{y_t^2}{2} \frac{M_P^2 h^2}{\xi_\chi \chi^2 + \xi_h h^2} = \frac{y_t^2}{2} v^2(\phi), \quad (4.23)$$

which is nothing else than the effective top mass in the Einstein frame. This corresponds to the prescription I in Ref. [10]. With this choice, the change in the shape of the potential is very small, given the insignificant variation of $v^2(\phi)$ during inflation. The change in the inflationary observables n_s and r is therefore expected to be completely negligible. A second possibility is to choose the scale invariant direction along the dilaton field χ

$$\mu_{\text{II}}^2 = \frac{\hat{\mu}^2}{M_P^2} \xi_\chi \chi^2. \quad (4.24)$$

The optimal choice of $\hat{\mu}$ in this prescription is

$$\hat{\mu}_{\text{II}}^2(\phi) = \frac{y_t^2}{2} \frac{M_P^2 h^2}{\xi_\chi \chi^2} = \frac{y_t^2}{2} v^2(\phi) \frac{1-\varsigma}{\varsigma \sinh^2(a\phi/M_P)}, \quad (4.25)$$

which, at the end of inflation, coincides with the effective top mass in the Jordan frame. This corresponds to the prescription II in Ref. [10]. Note that contrary to the previous case, this choice strongly depends on the value of the ϕ field and noticeable contributions to the inflationary parameters are expected.

The calculation proceeds now along the same lines as those in Ref. [10], using the tree level RG enhanced potential and the one loop correction. The addition of the two loop effective potential does not significantly modify the result. The numerical outcome for the two prescriptions is shown in Fig. 5. As expected, the inflationary observables computed with the first prescription coincide with the tree level result. The only effect of the quantum corrections is setting a minimal value for the Higgs mass. This turns out to be $m_H > m_{\text{min}}$, with $m_{\text{min}} \simeq 129.5 \pm 5$ GeV (for details on the latest calculations of this value see Ref. [51, 52]). After the end of inflation and preheating, the system is outside the scale-invariant region and the fields settle down to the minimum of the potential. From the expansion of the potential (2.27) around the background, it is clear that all the contributions to the effective action will be again suppressed by powers of the exponent $e^{-\gamma r/M_P}$, in addition to powers of M_P , not affecting therefore the predictions of the model concerning the DE equation of state (2.29). Taking into account the above results, we conclude that the quantum corrections computed with the prescription I do not modify the classical consistency relation (2.30) characterizing Higgs-Dilaton cosmology. On the other hand, the inflationary observables computed using the prescription II clearly differ from the tree level result, especially for Higgs masses close to the critical value m_{min} at large ξ_χ . Note that in this prescription, the recent observation of a light Higgs-like state [53, 54], together with the present bounds on the spectral tilt n_s [55], further restrain the allowed ξ_χ interval.

²⁰ The constants still run with typical euclidean momenta in the amplitudes.

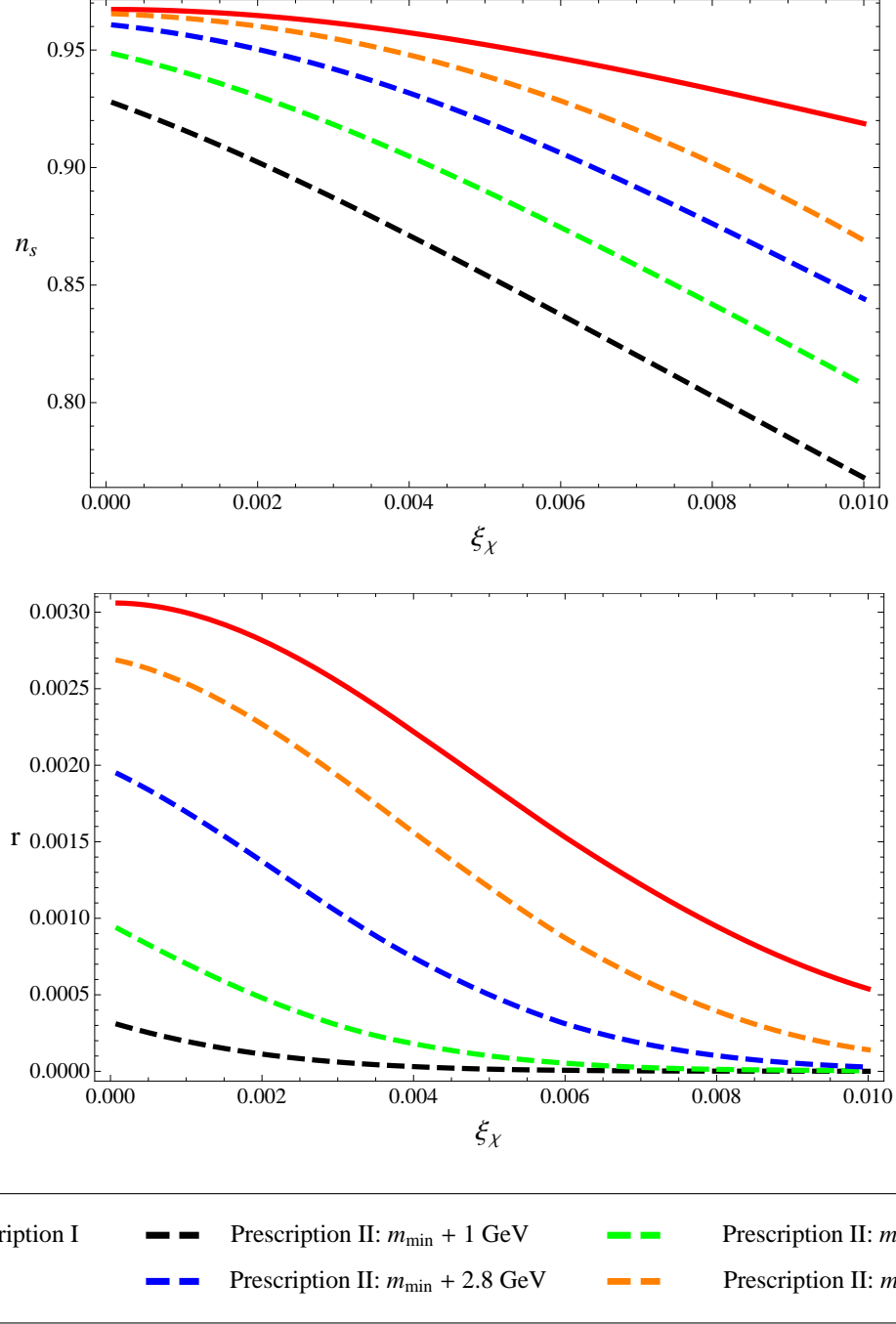


FIG. 5. The spectral index n_s (top) and tensor to scalar ratio r (bottom) as a function of the non-minimal coupling ξ_χ . The solid line corresponds to the quantization prescription I, which coincides with the tree level result. Dashed lines stand for the quantization choice II for different Higgs masses. The minimal Higgs boson mass m_{\min} can be obtained from Ref. [51].

V. CONCLUSIONS

The purpose of this paper was to study the self-consistency of the Higgs-Dilaton cosmological model. We determined the field-dependent UV cut-offs and studied their evolution in the different epochs throughout the history of the Universe. We showed that the cut-off value is higher than the relevant energy scales in the different periods, making the model a viable effective field theory describing inflation, reheating, and late-time acceleration of the Universe.

Since the theory is non-renormalizable, the loop expansion creates an infinite number of divergences, something that may challenge the classical predictions of the Higgs-Dilaton model. We argued that this is not the case if the UV-completion of the theory respects scale-invariance and the approximate shift symmetry for the dilaton field.

We computed within this framework the effective inflationary potential in the one-loop approximation and concluded that the dominant contribution comes from the chiral SM sector of the theory. We used two different regularizations prescriptions consistent with the symmetries of the model. In the “SI-prescription” of Ref. [35], with a field-dependent normalization point proportional to the effective Planck scale in the Jordan frame, the effective potential turns out to coincide with the tree level one. This leaves practically intact the consistency relation (2.30) which connects the inflationary spectral tilt to the deviation of the DE equation of state from a cosmological constant. This relation is however modified if the normalization point is chosen only along the dilaton’s direction, especially for Higgs masses near the critical value $m_{\min} \simeq 129.5 \pm 5$ GeV, which is amazingly close to the mass of the recently observed Higgs-like particle at the LHC [53, 54]. In the lack of a Planck scale UV completion, the proper choice of the normalization point μ can only be elucidated by improving the precision of the cosmological and particle physics observables.

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Appendix A

In this appendix we gather the Feynman rules as well as the expressions for the coefficients appearing in the one-loop diagrams in Section IV A. We denote with a dashed (solid) line the dilaton (Higgs) and perform the calculations in dimensional regularization in $D = 4 - 2\epsilon$ dimensions. After expanding the fields around their background values and normalizing the kinetic term for the dilaton, we find the following Feynman rules stemming from its kinetic term

$$\begin{array}{c} \diagup \text{---} \bullet \text{---} \text{---} \diagdown \\ \diagdown \text{---} \bullet \text{---} \text{---} \diagup \end{array} = \frac{a}{M_P} \tanh \left[\frac{a\bar{\phi}}{M_P} \right], \quad \begin{array}{c} \diagup \text{---} \bullet \text{---} \diagdown \\ \diagdown \text{---} \bullet \text{---} \diagup \end{array} = \frac{a^2}{2M_P^2} \left(1 + \tanh^2 \left[\frac{a\bar{\phi}}{M_P} \right] \right).$$

Using the above expression, we can calculate the coefficients appearing in the different diagrams. Let us start by considering the simplest diagram d_1 . We obtain

$$\begin{array}{c} \text{---} \text{---} \bullet \text{---} \text{---} \\ \text{---} \text{---} \bullet \text{---} \text{---} \end{array} = c_{1,1}^{d_1}(\bar{\phi}) \left(\frac{1}{\bar{\epsilon}} + f \right) \left(\frac{m_H}{M_P} \right)^2 (\partial r)^2,$$

with $1/\bar{\epsilon} = 1/\epsilon - \gamma + \log 4\pi$, and

$$c_{1,1}^{d_1}(\bar{\theta}) = \frac{a^2}{64\pi^2} \left(1 + \tanh^2 \left[\frac{a\bar{\theta}}{M_P} \right] \right), \quad f = -\log \left[\frac{m_H^2}{\mu^2} \right]. \quad (\text{A1})$$

Let us move to the more complicated diagram d_2 . We find

$$\begin{array}{c} \text{---} \bullet \text{---} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \text{---} \bullet \text{---} \end{array} = c_{1,2}^{d_2}(\bar{\phi}) \left[\left(\frac{1}{\bar{\epsilon}} + f' \right) \left(\frac{m_H}{M_P} \right)^2 + d \left(\frac{\partial}{M_P} \right)^2 \right] (\partial r)^2,$$

where

$$c_{1,2}^{d_2}(\bar{\theta}) = \frac{a^2}{16\pi^2} \tanh^2 \left[\frac{a\bar{\theta}}{M_P} \right], \quad f' = \frac{1}{2} - \log \left[\frac{m_H^2}{\mu^2} \right] \quad \text{and} \quad d = 0. \quad (\text{A2})$$

Note that in this particular diagram, the coefficient d is coincidentally zero. As we argued in Section IV A, this kind of terms are expected to appear by simple power-counting arguments in higher-loop diagrams. We see that in both diagrams, for the maximal value of the hyperbolic tangent, the corrections are suppressed by loop factors as well as powers of M_P .

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- [1] A. H. Guth, Phys. Rev. D **23** (1981) 347. A. D. Linde, Phys. Lett. B **108** (1982) 389. A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48** (1982) 1220. A. D. Linde, Phys. Lett. B **129** (1983) 177.
 - [2] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659** (2008) 703 [arXiv:0710.3755 [hep-th]].
 - [3] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, JCAP **0906** (2009) 029 [arXiv:0812.3622 [hep-ph]].
 - [4] J. Garcia-Bellido, D. G. Figueroa and J. Rubio, Phys. Rev. D **79** (2009) 063531 [arXiv:0812.4624 [hep-ph]].
 - [5] A. O. Barvinsky, A. Y. Kamenshchik and A. A. Starobinsky, JCAP **0811** (2008) 021 [arXiv:0809.2104 [hep-ph]].
 - [6] A. De Simone, M. P. Hertzberg and F. Wilczek, Phys. Lett. B **678** (2009) 1 [arXiv:0812.4946 [hep-ph]].
 - [7] F. L. Bezrukov, A. Magnin and M. Shaposhnikov, Phys. Lett. B **675** (2009) 88 [arXiv:0812.4950 [hep-ph]].
 - [8] C. P. Burgess, H. M. Lee and M. Trott, JHEP **0909** (2009) 103 [arXiv:0902.4465 [hep-ph]].
 - [9] J. L. F. Barbon and J. R. Espinosa, Phys. Rev. D **79** (2009) 081302 [arXiv:0903.0355 [hep-ph]].
 - [10] F. Bezrukov and M. Shaposhnikov, JHEP **0907** (2009) 089 [arXiv:0904.1537 [hep-ph]].
 - [11] A. O. Barvinsky, A. Y. Kamenshchik, C. Kiefer, A. A. Starobinsky and C. Steinwachs, JCAP **0912** (2009) 003 [arXiv:0904.1698 [hep-ph]].
 - [12] T. E. Clark, B. Liu, S. T. Love and T. ter Veldhuis, Phys. Rev. D **80** (2009) 075019 [arXiv:0906.5595 [hep-ph]].
 - [13] A. O. Barvinsky, A. Y. Kamenshchik, C. Kiefer, A. A. Starobinsky and C. F. Steinwachs, [arXiv:0910.1041 [hep-ph]].
 - [14] A. O. Barvinsky, A. Y. Kamenshchik, C. Kiefer and C. F. Steinwachs, Phys. Rev. D **81** (2010) 043530 [arXiv:0911.1408 [hep-th]].
 - [15] R. N. Lerner and J. McDonald, JCAP **1004** (2010) 015 [arXiv:0912.5463 [hep-ph]].
 - [16] C. P. Burgess, H. M. Lee and M. Trott, JHEP **1007** (2010) 007 [arXiv:1002.2730 [hep-ph]].
 - [17] M. P. Hertzberg, JHEP **1011** (2010) 023 [arXiv:1002.2995 [hep-ph]].
 - [18] R. N. Lerner and J. McDonald, Phys. Rev. D **82** (2010) 103525 [arXiv:1005.2978 [hep-ph]].
 - [19] G. F. Giudice and H. M. Lee, Phys. Lett. B **694** (2011) 294 [arXiv:1010.1417 [hep-ph]].
 - [20] R. N. Lerner and J. McDonald, JCAP **1211** (2012) 019 [arXiv:1112.0954 [hep-ph]].
 - [21] B. L. Spokoiny, Phys. Lett. B **147** (1984) 39.
 - [22] D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D **40** (1989) 1753.
 - [23] R. Fakir and W. G. Unruh, Phys. Rev. D **41** (1990) 1783.
 - [24] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, JHEP **1101** (2011) 016 [arXiv:1008.5157 [hep-ph]].
 - [25] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D **83** (2011) 025008 [arXiv:1008.2942 [hep-th]].
 - [26] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671** (2009) 187 [arXiv:0809.3395 [hep-th]].
 - [27] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84** (2011) 123504 [arXiv:1107.2163 [hep-ph]].
 - [28] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. **98** (2007) 021101 [hep-ph/0611184].
 - [29] J. L. Cervantes-Cota and H. Dehnen, Nucl. Phys. B **442** (1995) 391 [astro-ph/9505069].
 - [30] D. Blas, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84** (2011) 044001 [arXiv:1104.1392 [hep-th]].
 - [31] Y. Fujii and K. Maeda, Cambridge, USA: Univ. Pr. (2003) 240 p
 - [32] C. Wetterich, Nucl. Phys. B **302** (1988) 645.
 - [33] C. Wetterich, Nucl. Phys. B **302** (1988) 668.
 - [34] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37** (1988) 3406.
 - [35] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671** (2009) 162 [arXiv:0809.3406 [hep-th]].
 - [36] R. J. Scherrer and A. A. Sen, Phys. Rev. D **77** (2008) 083515 [arXiv:0712.3450 [astro-ph]].
 - [37] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10** (1974) 1145 [Erratum-ibid. D **11** (1975) 972].
 - [38] B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. Lett. **38** (1977) 883.
 - [39] B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D **16** (1977) 1519.
 - [40] M. S. Chanowitz, M. A. Furman and I. Hinchliffe, Phys. Lett. B **78** (1978) 285.
 - [41] M. S. Chanowitz, M. A. Furman and I. Hinchliffe, Nucl. Phys. B **153** (1979) 402.
 - [42] T. Appelquist and M. S. Chanowitz, Phys. Rev. Lett. **59** (1987) 2405 [Erratum-ibid. **60** (1988) 1589].
 - [43] J. Garcia-Bellido, J. Rubio and M. Shaposhnikov, Phys. Lett. B **718** (2012) 507 [arXiv:1209.2119 [hep-ph]].
 - [44] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, JCAP **1110** (2011) 001 [arXiv:1106.5019 [hep-ph]].
 - [45] M. E. Shaposhnikov and F. V. Tkachov, arXiv:0905.4857 [hep-th].
 - [46] F. Englert, C. Truffin and R. Gastmans, Nucl. Phys. B **117** (1976) 407.
 - [47] S. Dutta, K. Hagiwara, Q. -S. Yan and K. Yoshida, Nucl. Phys. B **790** (2008) 111 [arXiv:0705.2277 [hep-ph]].
 - [48] F. Feruglio, Int. J. Mod. Phys. A **8** (1993) 4937 [hep-ph/9301281].
 - [49] W. A. Bardeen, FERMILAB-CONF-95-391-T.

- [50] R. Jackiw, Phys. Rev. D **9** (1974) 1686.
- [51] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl and M. Shaposhnikov, JHEP **1210** (2012) 140 [arXiv:1205.2893 [hep-ph]].
- [52] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, JHEP **1208** (2012) 098 [arXiv:1205.6497 [hep-ph]].
- [53] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716** (2012) 30 [arXiv:1207.7235 [hep-ex]].
- [54] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716** (2012) 1 [arXiv:1207.7214 [hep-ex]].
- [55] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].